Matched Asymptotic Expansions of the Eigenvalues of a 3-D boundary-value problem relative to two cavities linked by a hole of small size

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In a lot of physical problems, the boundary of the computational domain is perforated. This configuration can lead to numerical difficulties when the diameter of the holes are really smaller than the other characteristic lengths. Indeed, it can be very costly to compute a sharp numerical approximation of the solution of such problems for two main reasons: With a standard method like finite elements or finite differences, a refined mesh cannot be avoided in the neighborhood of the hole; the mesh generation of a perforated structure can be a hard task.

Many authors have studied the effect of the perforation of the boundaries both from the theoretical and the numerical point of views, see for example [3–5]. However, less results have been obtained for the eigenvalue problem in the case of a three dimensional domain.

In [2], Gadyl'shin considered a two dimensional domain consisting of two domains linked by a small hole. He derived a complete asymptotic expansion of the scattering frequencies of the Laplacian operator equipped with Dirichlet boundary condition. In [1], these results were extended to the eigenvalues and eigenvectors of an elliptic operator with varying coefficients. In this talk, we are interested to a three dimensional configuration with varying coefficients and Neumann boundary condition.

References

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