HIGH-ORDER EXPLICIT LOCAL TIME-STEPPING FOR DAMPED WAVE EQUATIONS

Marcus Grote[1], Teodora Mitkova[2]

[1]University of Basel, Marcus.Grote@unibas.ch
[2]University of Basel, Teodora.Mitkova@unibas.ch

The accurate and reliable simulation of wave phenomena is of fundamental importance in a wide range of engineering applications. In the presence of complex geometry, adaptivity and mesh refinement are certainly key for the efficient numerical solution of the damped wave equation. Locally refined meshes, however, impose severe stability constraints on explicit time-stepping schemes due to the smallest elements in the mesh. When mesh refinement is restricted to a small region, the use of implicit methods, or a very small time step in the entire computational domain, are very high a price to pay. To overcome that stability restriction, local time-stepping methods are developed, which allow arbitrarily small time-steps precisely where small elements in the mesh are located. When combined with a finite element discretization in space with an essentially diagonal mass matrix, the resulting discrete numerical scheme is fully explicit.

Starting from the standard leap-frog scheme, explicit second-order local time-stepping integrators for transient wave motion have been derived [1, 2]. In the absence of damping these time-stepping schemes, when combined with the modified equation approach [3], yield methods of arbitrarily high (even) order. In the presence of damping, however, this approach cannot be used effectively. Therefore, we derive explicit local time-stepping schemes of arbitrarily high accuracy starting insted from explicit multi-step Adams-Bashforth methods. Numerical experiments validate the theoretical results and illustrate the efficiency of the proposed time integration schemes.

References

[1] J. Diaz, M.J. Grote, *Energy conserving explicit local time-stepping for second-order wave equations*, SIAM J. Sci. Comput., **31** (2009), No. 3, pp. 1985–2014.

[2] M.J. Grote, T. Mitkova, *Explicit local time-stepping for Maxwell's equations*, J. Comput. App. Math., doi:10.1016/j.cam.2010.04.028.

[3] G.R. Shubin, J.B. Bell, *A modified equation approach to constructing fourth order methods for acoustic wave propagation*, SIAM J. Sci. Stat. Comput., **8** (1987), pp. 135–151.